# The force hierarchy and geodynamo regimes of Earth's core


Rob Teed

University of Glasgow, Scotland


GFD seminar, Kyōto


Thanks to collaborators:
Emmanuel Dormy,
Ecole Normale Supérieure, Paris


University of Glasgow

Second oldest university in Japan (1897)

Second oldest university in Japan (1897)

Second oldest university in Scotland (1451)

# The Geomagnetic Field

Interest in the dynamics of Earth's core arises from its ability to generate the geomagnetic field.

# The Geomagnetic Field

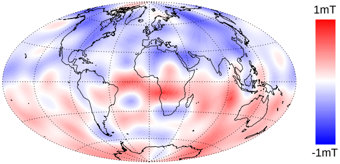
Interest in the dynamics of Earth's core arises from its ability to generate the geomagnetic field.

What might you already know about the geomagnetic field?

# The Geomagnetic Field

Interest in the dynamics of Earth's core arises from its ability to generate the geomagnetic field.

What might you already know about the geomagnetic field?

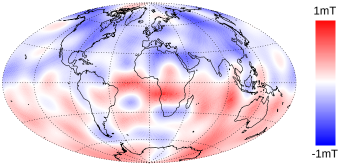



- Field is predominantly dipolar (but also note the patches of reversed flux found at high latitudes).

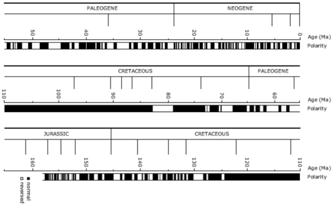
# The Geomagnetic Field

Interest in the dynamics of Earth's core arises from its ability to generate the geomagnetic field.

What might you already know about the geomagnetic field?

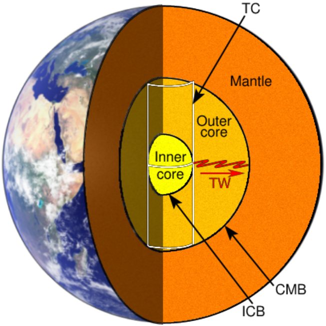



- Field is predominantly dipolar (but also note the patches of reversed flux found at high latitudes).




- Reversals of the field dipolarity occur (seemingly at random intervals and a reversal takes thousands of years to complete).

# Structure of Earth




ICB = Inner core boundary
CMB = Core-mantle boundary
TC = Tangent cylinder

- Fluid outer core is seat of dynamo giving rise to geomagnetic field.
- Convection arises from heat and light material released at inner core boundary.
- Magnetic field is continually replenished through induction (combining Faraday's law, Ampère's law, and Ohm's law)
- Twisting and stretching of field lines by chaotic convection generates electric current, in turn re-generating magnetic field.

# Observations vs. Simulations

Try to understand the generation of the geomagnetic field through observations and theory/simulations

# Observations vs. Simulations

Try to understand the generation of the geomagnetic field through observations and theory/simulations

Observations (from ground and satellites):

- It's the real thing! (and therefore the system parameters are Earth-like!)
- Sparse data and surface field can only be extended down to the core-mantle boundary
- Impossible to take measurements in the core itself $\rightarrow$ requirement for mathematical theory and computer simulations

# Observations vs. Simulations

Try to understand the generation of the geomagnetic field through observations and theory/simulations

Observations (from ground and satellites):

- It's the real thing! (and therefore the system parameters are Earth-like!)
- Sparse data and surface field can only be extended down to the core-mantle boundary
- Impossible to take measurements in the core itself $\rightarrow$ requirement for mathematical theory and computer simulations

Simulations:

- Offer full data set in the outer core
- Flow and magnetic field can be analysed directly
- Cannot reach the correct parameter regime for the Earth's core; large viscosity must be used to suppress unresolvable length scales

# Observations vs. Simulations

Try to understand the generation of the geomagnetic field through observations and theory/simulations

Observations (from ground and satellites):

- It's the real thing! (and therefore the system parameters are Earth-like!)
- Sparse data and surface field can only be extended down to the core-mantle boundary
- Impossible to take measurements in the core itself $\rightarrow$ requirement for mathematical theory and computer simulations

Simulations:

- Offer full data set in the outer core
- Flow and magnetic field can be analysed directly
- Cannot reach the correct parameter regime for the Earth's core; large viscosity must be used to suppress unresolvable length scales

Aim to match simulations to observations of the changing geomagnetic field thereby understanding dynamics in the core.

# Geodynamo simulations - physical setup

- Spherical polar coordinate system, $(r, \theta, \phi)$.
- Spherical shell radially bounded above at $r = r_{\rm o}$ by an electrically insulating mantle and below at $r = r_{\rm i}$ by an electrically insulating (or conducting) inner core.
- Rotates about the vertical ($z$-axis) with rotation rate $\Omega$ and gravity acts radially inward, $\mathbf{g} = g\mathbf{r}$.
- Boussinesq approximation used - density, $\rho$, treated as a constant except for the source of buoyancy
- Fluid is assumed to have constant values of $\rho$, $\nu$, $\kappa$ and $\eta$, the outer core density, kinematic viscosity, thermal diffusivity and magnetic diffusivity respectively.

# Geodynamo equations

Evolution equations for velocity, $\mathbf{u}$, temperature $T$, and magnetic field, $\mathbf{B}$:

$$\underbrace{\mathcal{E}_\mathrm{m}\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right)}_{\text{inertia (I)}} = \underbrace{-\nabla p}_{\text{pressure (P)}} \underbrace{-2\hat{\mathbf{z}}\times\mathbf{u}}_{\text{Coriolis (C)}} \underbrace{+(\nabla\times\mathbf{B})\times\mathbf{B}}_{\text{Lorentz (M)}} \underbrace{+\widetilde{Ra}T\mathbf{r}}_{\text{buoyancy (A)}} \underbrace{+\mathcal{E}\nabla^2\mathbf{u}}_{\text{viscous (V)}},$$

$$\frac{\partial T}{\partial t} + \mathbf{u}\cdot\nabla T = q\nabla^2 T,$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{u}\times\mathbf{B}) + \nabla^2\mathbf{B},$$

with conditions: $\nabla\cdot\mathbf{u} = 0$ and $\nabla\cdot\mathbf{B} = 0$.

# Geodynamo equations

Evolution equations for velocity, $\mathbf{u}$, temperature $T$, and magnetic field, $\mathbf{B}$:

$$\underbrace{\mathcal{E}_\mathrm{m}\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right)}_{\text{inertia (I)}} = \underbrace{-\nabla p}_{\text{pressure (P)}} \underbrace{-2\hat{\mathbf{z}}\times\mathbf{u}}_{\text{Coriolis (C)}} \underbrace{+(\nabla\times\mathbf{B})\times\mathbf{B}}_{\text{Lorentz (M)}} \underbrace{+\widetilde{Ra}T\mathbf{r}}_{\text{buoyancy (A)}} \underbrace{+\mathcal{E}\nabla^2\mathbf{u}}_{\text{viscous (V)}},$$

$$\frac{\partial T}{\partial t} + \mathbf{u}\cdot\nabla T = q\nabla^2 T,$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{u}\times\mathbf{B}) + \nabla^2\mathbf{B},$$

with conditions: $\nabla\cdot\mathbf{u} = 0$ and $\nabla\cdot\mathbf{B} = 0$.
4 key input parameters:

$$\mathcal{E} = \frac{\nu}{\Omega d^2}, \qquad [\mathcal{E}]_{\text{core}} \sim 10^{-15}, \qquad [\mathcal{E}]_{\text{sim}} \in [10^{-6}, 10^{-3}]$$

# Geodynamo equations

Evolution equations for velocity, $\mathbf{u}$, temperature $T$, and magnetic field, $\mathbf{B}$:

$$\underbrace{\mathcal{E}\mathrm{m}\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right)}_{\text{inertia (I)}} = \underbrace{-\nabla p}_{\text{pressure (P)}} \underbrace{-2\hat{\mathbf{z}}\times\mathbf{u}}_{\text{Coriolis (C)}} \underbrace{+(\nabla\times\mathbf{B})\times\mathbf{B}}_{\text{Lorentz (M)}} \underbrace{+\widetilde{Ra}T\mathbf{r}}_{\text{buoyancy (A)}} \underbrace{+\mathcal{E}\nabla^2\mathbf{u}}_{\text{viscous (V)}},$$

$$\frac{\partial T}{\partial t} + \mathbf{u}\cdot\nabla T = q\nabla^2 T,$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{u}\times\mathbf{B}) + \nabla^2\mathbf{B},$$

with conditions: $\nabla\cdot\mathbf{u} = 0$ and $\nabla\cdot\mathbf{B} = 0$.
4 key input parameters:

$$\mathcal{E} = \frac{\nu}{\Omega d^2}, \qquad [\mathcal{E}]_{\text{core}} \sim 10^{-15}, \qquad [\mathcal{E}]_{\text{sim}} \in [10^{-6}, 10^{-3}]$$

$$\mathcal{E}\mathrm{m} = \frac{\eta}{\Omega d^2}, \qquad [\mathcal{E}\mathrm{m}]_{\text{core}} \sim 10^{-9}, \qquad [\mathcal{E}\mathrm{m}]_{\text{sim}} \in [10^{-7}, 10^{-3}]$$

# Geodynamo equations

Evolution equations for velocity, $\mathbf{u}$, temperature $T$, and magnetic field, $\mathbf{B}$:

$$\underbrace{\mathcal{E}\mathrm{m}\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right)}_{\text{inertia (I)}} = \underbrace{-\nabla p}_{\text{pressure (P)}} \underbrace{-2\hat{\mathbf{z}}\times\mathbf{u}}_{\text{Coriolis (C)}} \underbrace{+(\nabla\times\mathbf{B})\times\mathbf{B}}_{\text{Lorentz (M)}} \underbrace{+\widetilde{Ra}T\mathbf{r}}_{\text{buoyancy (A)}} \underbrace{+\mathcal{E}\nabla^2\mathbf{u}}_{\text{viscous (V)}},$$

$$\frac{\partial T}{\partial t} + \mathbf{u}\cdot\nabla T = q\nabla^2 T,$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{u}\times\mathbf{B}) + \nabla^2\mathbf{B},$$

with conditions: $\nabla\cdot\mathbf{u} = 0$ and $\nabla\cdot\mathbf{B} = 0$.
4 key input parameters:

$$\mathcal{E} = \frac{\nu}{\Omega d^2}, \qquad [\mathcal{E}]_{\text{core}} \sim 10^{-15}, \qquad [\mathcal{E}]_{\text{sim}} \in [10^{-6}, 10^{-3}]$$

$$\mathcal{E}\mathrm{m} = \frac{\eta}{\Omega d^2}, \qquad [\mathcal{E}\mathrm{m}]_{\text{core}} \sim 10^{-9}, \qquad [\mathcal{E}\mathrm{m}]_{\text{sim}} \in [10^{-7}, 10^{-3}]$$

$$\widetilde{Ra} = \frac{\alpha g \Delta T d}{\Omega\eta}, \qquad [\widetilde{Ra}]_{\text{core}} \sim 10^{10} \qquad [\widetilde{Ra}]_{\text{sim}} \in [10 - 10^4]$$

# Geodynamo equations

Evolution equations for velocity, $\mathbf{u}$, temperature $T$, and magnetic field, $\mathbf{B}$:

$$\underbrace{\mathcal{E}\mathrm{m}\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right)}_{\text{inertia (I)}} = \underbrace{-\nabla p}_{\text{pressure (P)}} \underbrace{-2\hat{\mathbf{z}}\times\mathbf{u}}_{\text{Coriolis (C)}} \underbrace{+(\nabla\times\mathbf{B})\times\mathbf{B}}_{\text{Lorentz (M)}} \underbrace{+\widetilde{Ra}T\mathbf{r}}_{\text{buoyancy (A)}} \underbrace{+\mathcal{E}\nabla^2\mathbf{u}}_{\text{viscous (V)}},$$

$$\frac{\partial T}{\partial t} + \mathbf{u}\cdot\nabla T = q\nabla^2 T,$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{u}\times\mathbf{B}) + \nabla^2\mathbf{B},$$

with conditions: $\nabla\cdot\mathbf{u} = 0$ and $\nabla\cdot\mathbf{B} = 0$.
4 key input parameters:

$$\mathcal{E} = \frac{\nu}{\Omega d^2}, \qquad [\mathcal{E}]_{\text{core}} \sim 10^{-15}, \qquad [\mathcal{E}]_{\text{sim}} \in [10^{-6}, 10^{-3}]$$

$$\mathcal{E}\mathrm{m} = \frac{\eta}{\Omega d^2}, \qquad [\mathcal{E}\mathrm{m}]_{\text{core}} \sim 10^{-9}, \qquad [\mathcal{E}\mathrm{m}]_{\text{sim}} \in [10^{-7}, 10^{-3}]$$

$$\widetilde{Ra} = \frac{\alpha g \Delta T d}{\Omega\eta}, \qquad [\widetilde{Ra}]_{\text{core}} \sim 10^{10} \qquad [\widetilde{Ra}]_{\text{sim}} \in [10 - 10^4]$$

$$q = \frac{\kappa}{\eta}, \qquad [q]_{\text{core}} \sim 10^{-5} \qquad [q]_{\text{sim}} \in [0.1 - 10]$$

# Geodynamo equations

Evolution equations for velocity, $\mathbf{u}$, temperature $T$, and magnetic field, $\mathbf{B}$:

$$\underbrace{\mathcal{E}\mathrm{m}\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right)}_{\text{inertia (I)}} = \underbrace{-\nabla p}_{\text{pressure (P)}} \underbrace{-2\hat{\mathbf{z}}\times\mathbf{u}}_{\text{Coriolis (C)}} \underbrace{+(\nabla\times\mathbf{B})\times\mathbf{B}}_{\text{Lorentz (M)}} \underbrace{+\widetilde{Ra}T\mathbf{r}}_{\text{buoyancy (A)}} \underbrace{+\mathcal{E}\nabla^2\mathbf{u}}_{\text{viscous (V)}},$$

$$\frac{\partial T}{\partial t} + \mathbf{u}\cdot\nabla T = q\nabla^2 T,$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{u}\times\mathbf{B}) + \nabla^2\mathbf{B},$$

with conditions: $\nabla\cdot\mathbf{u} = 0$ and $\nabla\cdot\mathbf{B} = 0$.
4 key input parameters:

$$\mathcal{E} = \frac{\nu}{\Omega d^2}, \qquad [\mathcal{E}]_{\text{core}} \sim 10^{-15}, \qquad [\mathcal{E}]_{\text{sim}} \in [10^{-6}, 10^{-3}]$$

$$\mathcal{E}\mathrm{m} = \frac{\eta}{\Omega d^2}, \qquad [\mathcal{E}\mathrm{m}]_{\text{core}} \sim 10^{-9}, \qquad [\mathcal{E}\mathrm{m}]_{\text{sim}} \in [10^{-7}, 10^{-3}]$$

$$\widetilde{Ra} = \frac{\alpha g \Delta T d}{\Omega\eta}, \qquad [\widetilde{Ra}]_{\text{core}} \sim 10^{10} \qquad [\widetilde{Ra}]_{\text{sim}} \in [10 - 10^4]$$

$$q = \frac{\kappa}{\eta}, \qquad [q]_{\text{core}} \sim 10^{-5} \qquad [q]_{\text{sim}} \in [0.1 - 10]$$

- $Ra = \alpha g\Delta T d/\Omega\kappa = \widetilde{Ra}/q$ is an alternative Rayleigh number (useful to relate to non-magnetic problem).

# Geodynamo equations

Evolution equations for velocity, $\mathbf{u}$, temperature $T$, and magnetic field, $\mathbf{B}$:

$$\underbrace{\mathcal{E}\mathrm{m}\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right)}_{\text{inertia (I)}} = \underbrace{-\nabla p}_{\text{pressure (P)}} \underbrace{-2\hat{\mathbf{z}}\times\mathbf{u}}_{\text{Coriolis (C)}} \underbrace{+(\nabla\times\mathbf{B})\times\mathbf{B}}_{\text{Lorentz (M)}} \underbrace{+\widetilde{Ra}T\mathbf{r}}_{\text{buoyancy (A)}} \underbrace{+\mathcal{E}\nabla^2\mathbf{u}}_{\text{viscous (V)}},$$

$$\frac{\partial T}{\partial t} + \mathbf{u}\cdot\nabla T = q\nabla^2 T,$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{u}\times\mathbf{B}) + \nabla^2\mathbf{B},$$

with conditions: $\nabla\cdot\mathbf{u} = 0$ and $\nabla\cdot\mathbf{B} = 0$.
4 key input parameters:

$$\mathcal{E} = \tfrac{\nu}{\Omega d^2}, \qquad [\mathcal{E}]_{\text{core}} \sim 10^{-15}, \qquad [\mathcal{E}]_{\text{sim}} \in [10^{-6}, 10^{-3}]$$

$$\mathcal{E}\mathrm{m} = \tfrac{\eta}{\Omega d^2}, \qquad [\mathcal{E}\mathrm{m}]_{\text{core}} \sim 10^{-9}, \qquad [\mathcal{E}\mathrm{m}]_{\text{sim}} \in [10^{-7}, 10^{-3}]$$

$$\widetilde{Ra} = \tfrac{\alpha g \Delta T d}{\Omega\eta}, \qquad [\widetilde{Ra}]_{\text{core}} \sim 10^{10} \qquad [\widetilde{Ra}]_{\text{sim}} \in [10 - 10^4]$$

$$q = \tfrac{\kappa}{\eta}, \qquad [q]_{\text{core}} \sim 10^{-5} \qquad [q]_{\text{sim}} \in [0.1 - 10]$$

- $Ra = \alpha g\Delta T d/\Omega\kappa = \widetilde{Ra}/q$ is an alternative Rayleigh number (useful to relate to non-magnetic problem).
- Often use $\widetilde{Ra}' = Ra/Ra_c = \widetilde{Ra}/qRa_c$, as a measure of supercriticality. $Ra_c$ is the critical Rayleigh number for the onset of (non-magnetic) convection.

# Approaches to geodynamo simulations

How should we approach studies using simulations given we cannot use the correct input parameter values?

# Approaches to geodynamo simulations

How should we approach studies using simulations given we cannot use the correct input parameter values?

Possible options:

1. 'Correct' parameter space: aim to move the parameters as close to their physical values as possible.

# Approaches to geodynamo simulations

How should we approach studies using simulations given we cannot use the correct input parameter values?

Possible options:

1. ‘Correct’ parameter space: aim to move the parameters as close to their physical values as possible.

$\rightarrow$ Can have unintended outcomes such as non-dipolar solutions, solutions with a weak magnetic field, etc.

# Approaches to geodynamo simulations

How should we approach studies using simulations given we cannot use the correct input parameter values?

Possible options:

1. 'Correct' parameter space: aim to move the parameters as close to their physical values as possible.

→ Can have unintended outcomes such as non-dipolar solutions, solutions with a weak magnetic field, etc.

2. Correct solution space: aim to find solutions with the expected *balance of forces* within the momentum equation by performing parameter sweeps

# Approaches to geodynamo simulations

How should we approach studies using simulations given we cannot use the correct input parameter values?

Possible options:

1 'Correct' parameter space: aim to move the parameters as close to their physical values as possible.

→ Can have unintended outcomes such as non-dipolar solutions, solutions with a weak magnetic field, etc.

2 Correct solution space: aim to find solutions with the expected *balance of forces* within the momentum equation by performing parameter sweeps

→ Allows for the identification of suitable parameter regimes despite input parameters not close to Earth-like values.
Then preserve the force balance by moving all parameters towards Earth-like values in a systematic way.

# Force balances

- Forces acting in the non-dimensionalised system are:

$$\mathbf{F}_{\mathsf{I}} = \mathcal{E}_{\mathrm{m}} \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) \qquad \mathbf{F}_{\mathsf{P}} = -\nabla p \qquad \mathbf{F}_{\mathsf{C}} = -2\hat{\mathbf{z}} \times \mathbf{u}$$

$$\mathbf{F}_{\mathsf{M}} = (\nabla \times \mathbf{B}) \times \mathbf{B} \qquad \mathbf{F}_{\mathsf{A}} = \widetilde{Ra} T \mathbf{r} \qquad \mathbf{F}_{\mathsf{V}} = \mathcal{E} \nabla^2 \mathbf{u}$$

# Force balances

- Forces acting in the non-dimensionalised system are:

$$\mathbf{F}_{\mathsf{I}} = \mathcal{E}_{\mathrm{m}} \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) \qquad \mathbf{F}_{\mathsf{P}} = -\nabla p \qquad \mathbf{F}_{\mathsf{C}} = -2\hat{\mathbf{z}} \times \mathbf{u}$$

$$\mathbf{F}_{\mathsf{M}} = (\nabla \times \mathbf{B}) \times \mathbf{B} \qquad \mathbf{F}_{\mathsf{A}} = \widetilde{Ra} T \mathbf{r} \qquad \mathbf{F}_{\mathsf{V}} = \mathcal{E} \nabla^2 \mathbf{u}$$

- Notable balances:

# Force balances

- Forces acting in the non-dimensionalised system are:

$$\mathbf{F}_\mathsf{I} = \mathcal{E}_\mathrm{m}\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) \qquad \mathbf{F}_\mathsf{P} = -\nabla p \qquad \mathbf{F}_\mathsf{C} = -2\hat{\mathbf{z}}\times\mathbf{u}$$

$$\mathbf{F}_\mathsf{M} = (\nabla\times\mathbf{B})\times\mathbf{B} \qquad \mathbf{F}_\mathsf{A} = \widetilde{Ra}T\mathbf{r} \qquad \mathbf{F}_\mathsf{V} = \mathcal{E}\nabla^2\mathbf{u}$$

- Notable balances:
  - Geostrophic balance: $\mathbf{F}_\mathsf{P} = \mathbf{F}_\mathsf{C}$. For no magnetic field and no convection, (and small $\mathcal{E}$ and $\mathcal{E}_\mathrm{m}$). Curl $\Rightarrow \partial\mathbf{u}/\partial z = \mathbf{0}$ (Taylor-Proudman constraint leading to $z$-independent motion - classical for (rapidly) rotating fluids and independent of pressure)

# Force balances

- Forces acting in the non-dimensionalised system are:

$$\mathbf{F}_\mathsf{I} = \mathcal{E}_\mathrm{m}\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) \qquad \mathbf{F}_\mathsf{P} = -\nabla p \qquad \mathbf{F}_\mathsf{C} = -2\hat{\mathbf{z}}\times\mathbf{u}$$

$$\mathbf{F}_\mathsf{M} = (\nabla\times\mathbf{B})\times\mathbf{B} \qquad \mathbf{F}_\mathsf{A} = \widetilde{Ra}T\mathbf{r} \qquad \mathbf{F}_\mathsf{V} = \mathcal{E}\nabla^2\mathbf{u}$$

- Notable balances:
  - Geostrophic balance: $\mathbf{F}_\mathsf{P} = \mathbf{F}_\mathsf{C}$. For no magnetic field and no convection, (and small $\mathcal{E}$ and $\mathcal{E}_\mathrm{m}$). Curl $\Rightarrow \partial\mathbf{u}/\partial z = \mathbf{0}$ (Taylor-Proudman constraint leading to $z$-independent motion - classical for (rapidly) rotating fluids and independent of pressure)
  - 'Quasi'-geostrophic ('QG') balance(s), $\mathbf{F}_\mathsf{P} = \mathbf{F}_\mathsf{C}$ $(+\mathbf{F}_\mathsf{M}?)$ $(+\mathbf{F}_\mathsf{A}?)$ $(+\mathbf{F}_\mathsf{I}?)$ Curl $\Rightarrow \partial\mathbf{u}/\partial z \sim \mathbf{0}?$ (loosening of Taylor-Proudman constraint)

# Force balances

- Forces acting in the non-dimensionalised system are:

$$\mathbf{F}_\mathsf{I} = \mathcal{E}_\mathrm{m}\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) \qquad \mathbf{F}_\mathsf{P} = -\nabla p \qquad \mathbf{F}_\mathsf{C} = -2\hat{\mathbf{z}}\times\mathbf{u}$$

$$\mathbf{F}_\mathsf{M} = (\nabla\times\mathbf{B})\times\mathbf{B} \qquad \mathbf{F}_\mathsf{A} = \widetilde{Ra}T\mathbf{r} \qquad \mathbf{F}_\mathsf{V} = \mathcal{E}\nabla^2\mathbf{u}$$

- Notable balances:
  - Geostrophic balance: $\mathbf{F}_\mathsf{P} = \mathbf{F}_\mathsf{C}$. For no magnetic field and no convection, (and small $\mathcal{E}$ and $\mathcal{E}_\mathrm{m}$).
    Curl $\Rightarrow \partial\mathbf{u}/\partial z = \mathbf{0}$ (Taylor-Proudman constraint leading to $z$-independent motion - classical for (rapidly) rotating fluids and independent of pressure)
  - 'Quasi'-geostrophic ('QG') balance(s), $\mathbf{F}_\mathsf{P} = \mathbf{F}_\mathsf{C}$ $(+\mathbf{F}_\mathsf{M}?)$ $(+\mathbf{F}_\mathsf{A}?)$ $(+\mathbf{F}_\mathsf{I}?)$
    Curl $\Rightarrow \partial\mathbf{u}/\partial z \sim \mathbf{0}$? (loosening of Taylor-Proudman constraint)
  - MAC balance, $\mathbf{F}_\mathsf{M} \sim \mathbf{F}_\mathsf{A} \sim \mathbf{F}_\mathsf{C}$. Expected balance for Earth's core (aside from pressure gradient contributions...see later!).
    Curl $\Rightarrow \partial\mathbf{u}/\partial z \neq \mathbf{0}$ (broken Taylor-Proudman constraint)

# Force balances

- Forces acting in the non-dimensionalised system are:

$$\mathbf{F}_\mathsf{I} = \mathcal{E}_\mathrm{m}\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) \qquad \mathbf{F}_\mathsf{P} = -\nabla p \qquad \mathbf{F}_\mathsf{C} = -2\hat{\mathbf{z}}\times\mathbf{u}$$

$$\mathbf{F}_\mathsf{M} = (\nabla\times\mathbf{B})\times\mathbf{B} \qquad \mathbf{F}_\mathsf{A} = \widetilde{Ra}T\mathbf{r} \qquad \mathbf{F}_\mathsf{V} = \mathcal{E}\nabla^2\mathbf{u}$$

- Notable balances:
  - Geostrophic balance: $\mathbf{F}_\mathsf{P} = \mathbf{F}_\mathsf{C}$. For no magnetic field and no convection, (and small $\mathcal{E}$ and $\mathcal{E}_\mathrm{m}$). Curl $\Rightarrow \partial\mathbf{u}/\partial z = \mathbf{0}$ (Taylor-Proudman constraint leading to $z$-independent motion - classical for (rapidly) rotating fluids and independent of pressure)
  - 'Quasi'-geostrophic ('QG') balance(s), $\mathbf{F}_\mathsf{P} = \mathbf{F}_\mathsf{C}$ $(+\mathbf{F}_\mathsf{M}?)$ $(+\mathbf{F}_\mathsf{A}?)$ $(+\mathbf{F}_\mathsf{I}?)$ Curl $\Rightarrow \partial\mathbf{u}/\partial z \sim \mathbf{0}?$ (loosening of Taylor-Proudman constraint)
  - MAC balance, $\mathbf{F}_\mathsf{M} \sim \mathbf{F}_\mathsf{A} \sim \mathbf{F}_\mathsf{C}$. Expected balance for Earth's core (aside from pressure gradient contributions...see later!). Curl $\Rightarrow \partial\mathbf{u}/\partial z \neq \mathbf{0}$ (broken Taylor-Proudman constraint)
  - VAC balance, $\mathbf{F}_\mathsf{V} \sim \mathbf{F}_\mathsf{A} \sim \mathbf{F}_\mathsf{C}$. Close to onset of convection/dynamo action - actually preserves geostrophy due to form of V and A.

# Force balances

- Forces acting in the non-dimensionalised system are:

$$\mathbf{F}_{\mathsf{I}} = \mathcal{E}_{\mathrm{m}}\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) \qquad \mathbf{F}_{\mathsf{P}} = -\nabla p \qquad \mathbf{F}_{\mathsf{C}} = -2\hat{\mathbf{z}}\times\mathbf{u}$$

$$\mathbf{F}_{\mathsf{M}} = (\nabla\times\mathbf{B})\times\mathbf{B} \qquad \mathbf{F}_{\mathsf{A}} = \widetilde{Ra}T\mathbf{r} \qquad \mathbf{F}_{\mathsf{V}} = \mathcal{E}\nabla^2\mathbf{u}$$

- Notable balances:
  - Geostrophic balance: $\mathbf{F}_{\mathsf{P}} = \mathbf{F}_{\mathsf{C}}$. For no magnetic field and no convection, (and small $\mathcal{E}$ and $\mathcal{E}_{\mathrm{m}}$). Curl $\Rightarrow \partial\mathbf{u}/\partial z = \mathbf{0}$ (Taylor-Proudman constraint leading to $z$-independent motion - classical for (rapidly) rotating fluids and independent of pressure)
  - 'Quasi'-geostrophic ('QG') balance(s), $\mathbf{F}_{\mathsf{P}} = \mathbf{F}_{\mathsf{C}}$ $(+\mathbf{F}_{\mathsf{M}}?)$ $(+\mathbf{F}_{\mathsf{A}}?)$ $(+\mathbf{F}_{\mathsf{I}}?)$
    Curl $\Rightarrow \partial\mathbf{u}/\partial z \sim \mathbf{0}?$ (loosening of Taylor-Proudman constraint)
  - MAC balance, $\mathbf{F}_{\mathsf{M}} \sim \mathbf{F}_{\mathsf{A}} \sim \mathbf{F}_{\mathsf{C}}$. Expected balance for Earth's core (aside from pressure gradient contributions...see later!).
    Curl $\Rightarrow \partial\mathbf{u}/\partial z \neq \mathbf{0}$ (broken Taylor-Proudman constraint)
  - VAC balance, $\mathbf{F}_{\mathsf{V}} \sim \mathbf{F}_{\mathsf{A}} \sim \mathbf{F}_{\mathsf{C}}$. Close to onset of convection/dynamo action - actually preserves geostrophy due to form of V and A.
  - CIA balance, $\mathbf{F}_{\mathsf{C}} \sim \mathbf{F}_{\mathsf{I}} \sim \mathbf{F}_{\mathsf{A}}$. At high convective driving and weak magnetic field.

# Force balances

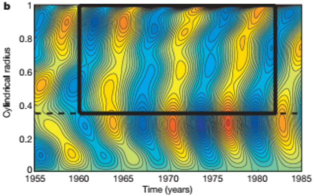
- Seen geostrophic, MAC, VAC, etc.

# Force balances

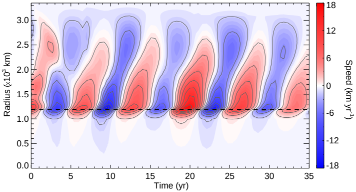
- Seen geostrophic, MAC, VAC, etc.
- MAC balance gives rise to torsional/magnetic Rossby waves as found in simulations (e.g. Teed+, 2014; Hori+, 2015)

# Force balances

- Seen geostrophic, MAC, VAC, etc.
- MAC balance gives rise to torsional/magnetic Rossby waves as found in simulations (e.g. Teed+, 2014; Hori+, 2015)



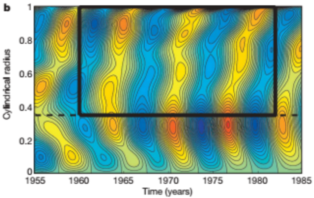

*Observation (Gillet+, 2010)*



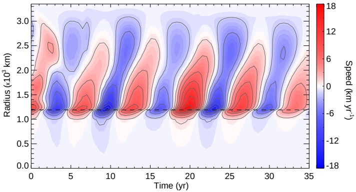

*Simulation (Teed+, 2014)*

# Force balances

- Seen geostrophic, MAC, VAC, etc.
- MAC balance gives rise to torsional/magnetic Rossby waves as found in simulations (e.g. Teed+, 2014; Hori+, 2015)




*Observation (Gillet+, 2010)*

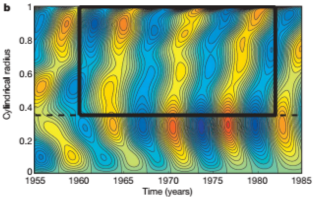



*Simulation (Teed+, 2014)*

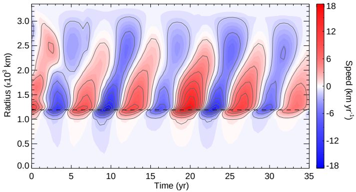
- Observations of waves suggest Earth in MAC regime

# Force balances

- Seen geostrophic, MAC, VAC, etc.
- MAC balance gives rise to torsional/magnetic Rossby waves as found in simulations (e.g. Teed+, 2014; Hori+, 2015)




*Observation (Gillet+, 2010)*




*Simulation (Teed+, 2014)*

- Observations of waves suggest Earth in MAC regime
- Some previous investigations of force balances:
    - Rotvig & Jones, Phys Rev E, 2002
    - Soderlund+, PEPS, 2015
    - Yadav+, PNAS, 2016
    - Schaeffer+, GJI, 2017
    - Schwaiger+, GJI, 2019, 2021
    - Teed & Dormy, JFM, 2023

# Lengthscale dependent forces

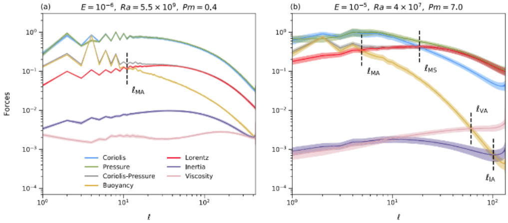
- Analysis of the hierarchy and balance of forces is complicated by:
  1. forces depend on position, lengthscale, and time
  2. a ‘zeroth order’ balance between the dynamically unimportant pressure gradient force and another force(s)

# Lengthscale dependent forces

- Analysis of the hierarchy and balance of forces is complicated by:
  1. forces depend on position, lengthscale, and time
  2. a ‘zeroth order’ balance between the dynamically unimportant pressure gradient force and another force(s)
- Studies have used ‘ageostrophic Coriolis force’: $\mathbf{F}_\mathsf{C}^\mathsf{ag} = \mathbf{F}_\mathsf{C} - \mathbf{F}_\mathsf{P}$ in attempt to address point 2 (Schwaiger+, GJI, 2019, 2021).

# Lengthscale dependent forces

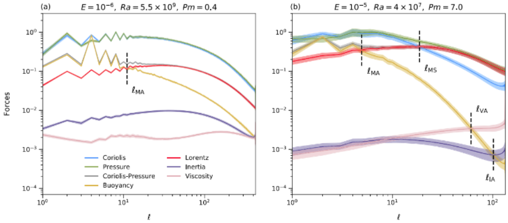
- Analysis of the hierarchy and balance of forces is complicated by:
  1. forces depend on position, lengthscale, and time
  2. a 'zeroth order' balance between the dynamically unimportant pressure gradient force and another force(s)
- Studies have used 'ageostrophic Coriolis force': $\mathbf{F}_\mathsf{C}^\mathsf{ag} = \mathbf{F}_\mathsf{C} - \mathbf{F}_\mathsf{P}$ in attempt to address point 2 (Schwaiger+, GJI, 2019, 2021).
- Leads to 'QG-MAC' regime: zeroth order 'QG' and first order MAC (C being $\mathbf{F}_\mathsf{C}^\mathsf{ag}$).




*Schwaiger+, 2021*

# Lengthscale dependent forces

- Analysis of the hierarchy and balance of forces is complicated by:
  1. forces depend on position, lengthscale, and time
  2. a 'zeroth order' balance between the dynamically unimportant pressure gradient force and another force(s)
- Studies have used 'ageostrophic Coriolis force': $\mathbf{F}_{\mathrm{C}}^{\mathrm{ag}} = \mathbf{F}_{\mathrm{C}} - \mathbf{F}_{\mathrm{P}}$ in attempt to address point 2 (Schwaiger+, GJI, 2019, 2021).
- Leads to 'QG-MAC' regime: zeroth order 'QG' and first order MAC (C being $\mathbf{F}_{\mathrm{C}}^{\mathrm{ag}}$).

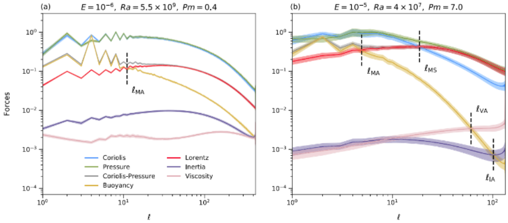



*Schwaiger+, 2021*

- Potentially works ok if the zeroth order balance is geostrophic everywhere

# Lengthscale dependent forces

- Analysis of the hierarchy and balance of forces is complicated by:
  1. forces depend on position, lengthscale, and time
  2. a 'zeroth order' balance between the dynamically unimportant pressure gradient force and another force(s)
- Studies have used 'ageostrophic Coriolis force': $\mathbf{F}_{\mathrm{C}}^{\mathrm{ag}} = \mathbf{F}_{\mathrm{C}} - \mathbf{F}_{\mathrm{P}}$ in attempt to address point 2 (Schwaiger+, GJI, 2019, 2021).
- Leads to 'QG-MAC' regime: zeroth order 'QG' and first order MAC (C being $\mathbf{F}_{\mathrm{C}}^{\mathrm{ag}}$).




*Schwaiger+, 2021*

- Potentially works ok if the zeroth order balance is geostrophic everywhere
- But that is not the case!

# Solenoidal forces

- Ageostrophic Coriolis force only relates to gradient parts of $\mathbf{F}_{\mathrm{P}}$ and $\mathbf{F}_{\mathrm{C}}$

# Solenoidal forces

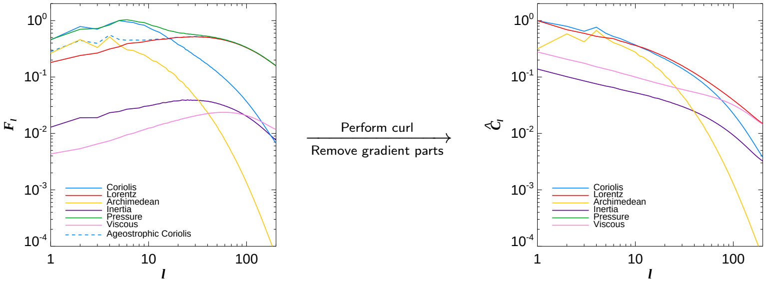
- Ageostrophic Coriolis force only relates to gradient parts of $\mathbf{F}_\mathrm{P}$ and $\mathbf{F}_\mathrm{C}$
- But **all** forces potentially have gradient parts that are not important for dynamics

# Solenoidal forces

- Ageostrophic Coriolis force only relates to gradient parts of $\mathbf{F}_{\mathsf{P}}$ and $\mathbf{F}_{\mathsf{C}}$
- But **all** forces potentially have gradient parts that are not important for dynamics
- Teed & Dormy (2023) proposed forming '**solenoidal forces**' by directly eliminating gradient parts of all forces to observe the important first order balance (MAC, VAC, etc.) directly

# Solenoidal forces

- Ageostrophic Coriolis force only relates to gradient parts of $\mathbf{F}_\mathrm{P}$ and $\mathbf{F}_\mathrm{C}$
- But **all** forces potentially have gradient parts that are not important for dynamics
- Teed & Dormy (2023) proposed forming '**solenoidal forces**' by directly eliminating gradient parts of all forces to observe the important first order balance (MAC, VAC, etc.) directly
- $\mathbf{F} = \nabla \times \mathbf{A} + \nabla\varphi$; eliminate $\nabla\varphi$ by:
  - *curling* $\mathbf{F}$. (Note: Taylor-Proudman constraint is formed this way!)
  - *projecting of forces* onto their solenoidal part: $\nabla \times \mathbf{A}$



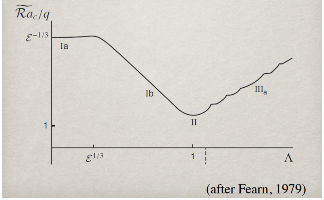

*Teed & Dormy, 2023*

# Weak and strong field branches - theory

The Elsasser number, $\Lambda$, is used as a measure of the field strength ($\Lambda$ is measure of Lorentz/Coriolis but has various definitions).

# Weak and strong field branches - theory

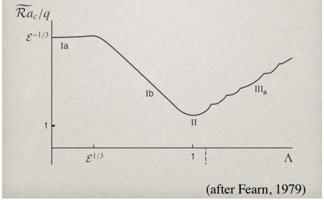
The Elsasser number, $\Lambda$, is used as a measure of the field strength ($\Lambda$ is measure of Lorentz/Coriolis but has various definitions).

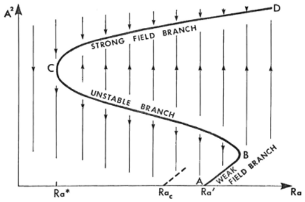



- Studies of magnetoconvection: *weak* magnetic field inhibits the formation of convection
- Fearn, 1979; once $\Lambda \sim O(\mathcal{E}^{1/3})$, magnetic field begins to *aid* convection

# Weak and strong field branches - theory

The Elsasser number, $\Lambda$, is used as a measure of the field strength ($\Lambda$ is measure of Lorentz/Coriolis but has various definitions).

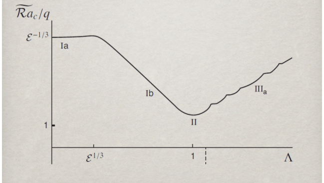



- Studies of magnetoconvection: *weak* magnetic field inhibits the formation of convection
- Fearn, 1979; once $\Lambda \sim O(\mathcal{E}^{1/3})$, magnetic field begins to *aid* convection
- Roberts, 1979; suggestion of a subcritical bifurcation with stable weak and strong field branches (once $\Lambda \sim O(1)$)

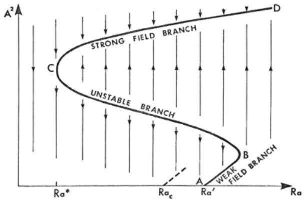
# Weak and strong field branches - theory

The Elsasser number, $\Lambda$, is used as a measure of the field strength ($\Lambda$ is measure of Lorentz/Coriolis but has various definitions).




(after Fearn, 1979)

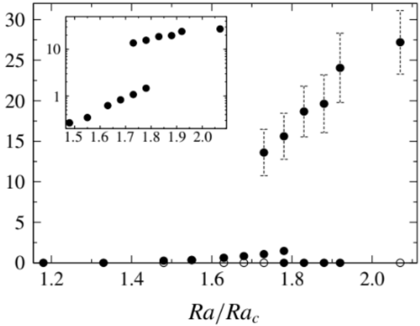
- Studies of magnetoconvection: *weak* magnetic field inhibits the formation of convection
- Fearn, 1979; once $\Lambda \sim O(\mathcal{E}^{1/3})$, magnetic field begins to *aid* convection




- Roberts, 1979; suggestion of a subcritical bifurcation with stable weak and strong field branches (once $\Lambda \sim O(1)$)
- Potential bistability - different regimes at the same input parameters

# Weak and strong field branches - simulations

- Dormy, 2016; identification of strong field branch and bistability in DNS at $\mathcal{E} = 3 \times 10^{-4}$ and $\mathcal{E}_\mathrm{m} = 1.7 \times 10^{-5}$




- Requires $\mathcal{E}_\mathrm{m}$ to be chosen within a ‘sweet-spot’ range of values (dependent on $\mathcal{E}$)...

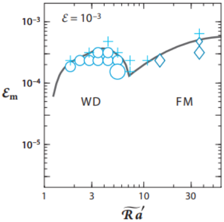
# Regime diagrams

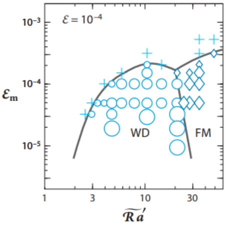
Each plot is decreasing $\mathcal{E}$ →

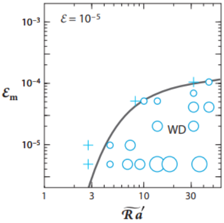
← Decreasing $\mathcal{E}_{\mathrm{m}}$

Solutions become non-magnetic ↑

Solutions become non-dipolar ↓

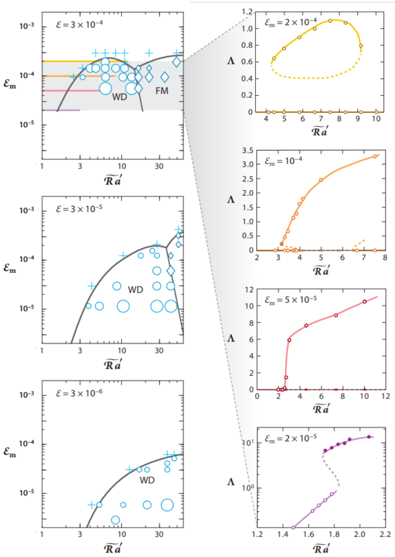



Solutions become non-dipolar with increasing $\widetilde{Ra}'$ →

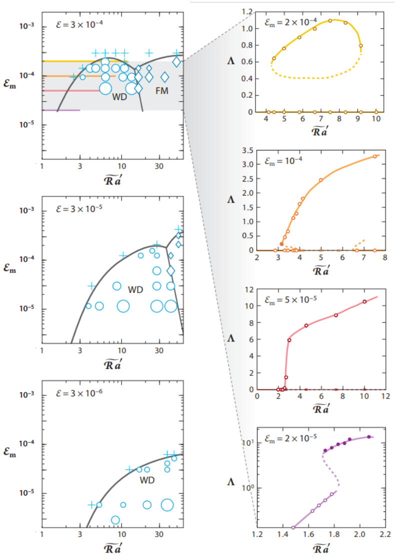
# Bifurcation diagrams




*Dormy, 2025*

- Bifurcation diagrams for $\mathcal{E} = 3 \times 10^{-4}$

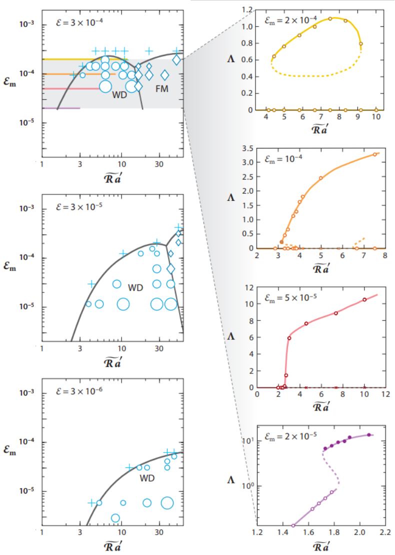
# Bifurcation diagrams




*Dormy, 2025*

- Bifurcation diagrams for $\mathcal{E} = 3 \times 10^{-4}$
- Diagrams differ (isola, supercritical, subcritical bifurcations) as $\mathcal{E}_m$ is varied

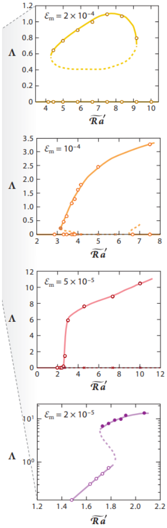
# Bifurcation diagrams



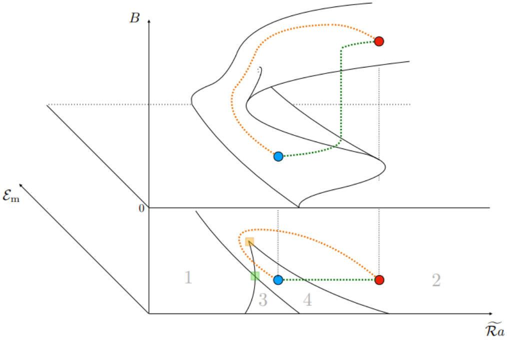

*Dormy, 2025*

- Bifurcation diagrams for $\mathcal{E} = 3 \times 10^{-4}$
- Diagrams differ (isola, supercritical, subcritical bifurcations) as $\mathcal{E}_m$ is varied
- Weak-strong branching is found at low enough $\mathcal{E}_m$

# Bifurcation diagrams



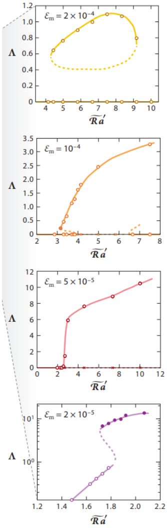

*Dormy, 2025*



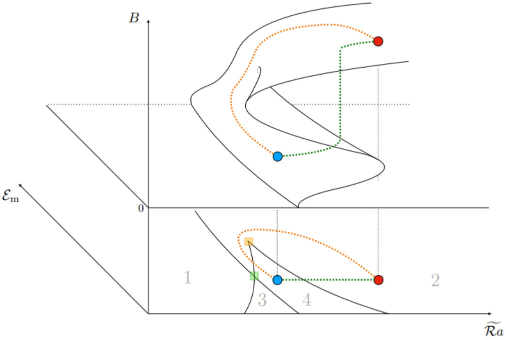

*Dormy, 2025*

- Tentative 3D bifurcation diagram for fixed $\mathcal{E}$

# Bifurcation diagrams



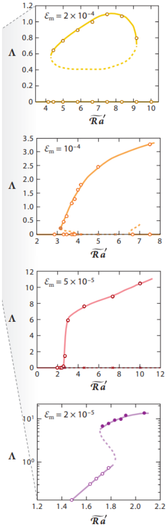

*Dormy, 2025*



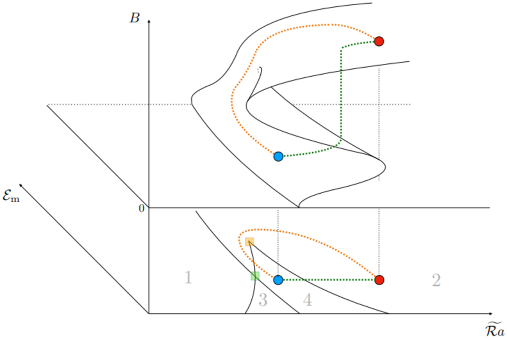

*Dormy, 2025*

- Tentative 3D bifurcation diagram for fixed $\mathcal{E}$
- Weak-strong branching occurs via a cusp singularity

# Bifurcation diagrams




*Dormy, 2025*




*Dormy, 2025*

- Tentative 3D bifurcation diagram for fixed $\mathcal{E}$
- Weak-strong branching occurs via a cusp singularity
- Supercritical branch exhibits a sharp step announcing the cusp

# Distinguished limit

- System of equations has four (main) control parameters ($\mathcal{E}$, $\mathcal{E}_{\mathrm{m}}$, $q$, $\widetilde{Ra}$)...

# Distinguished limit

- System of equations has four (main) control parameters ($\mathcal{E}$, $\mathcal{E}_{\mathrm{m}}$, $q$, $\widetilde{Ra}$)...
- ...three of which become very small at Earth-like values ($\mathcal{E} \sim 10^{-15}$, $\mathcal{E}_{\mathrm{m}} \sim 10^{-9}$, $q \sim 10^{-5}$)

# Distinguished limit

- System of equations has four (main) control parameters ($\mathcal{E}$, $\mathcal{E}_{\mathrm{m}}$, $q$, $\widetilde{Ra}$)...
- ...three of which become very small at Earth-like values ($\mathcal{E} \sim 10^{-15}$, $\mathcal{E}_{\mathrm{m}} \sim 10^{-9}$, $q \sim 10^{-5}$)
- Unclear how to choose $\mathcal{E}_{\mathrm{m}}$ (and $q$) for a given $\mathcal{E}$...

# Distinguished limit

- System of equations has four (main) control parameters ($\mathcal{E}$, $\mathcal{E}_{\rm m}$, $q$, $\widetilde{Ra}$)...
- ...three of which become very small at Earth-like values ($\mathcal{E} \sim 10^{-15}$, $\mathcal{E}_{\rm m} \sim 10^{-9}$, $q \sim 10^{-5}$)
- Unclear how to choose $\mathcal{E}_{\rm m}$ (and $q$) for a given $\mathcal{E}$...
- ...but we could relate $\mathcal{E}$ and $\mathcal{E}_{\rm m}$ via a *distinguished limit* (Dormy, 2016)

# Distinguished limit

- System of equations has four (main) control parameters ($\mathcal{E}$, $\mathcal{E}_{\mathrm{m}}$, $q$, $\widetilde{Ra}$)...
- ...three of which become very small at Earth-like values ($\mathcal{E} \sim 10^{-15}$, $\mathcal{E}_{\mathrm{m}} \sim 10^{-9}$, $q \sim 10^{-5}$)
- Unclear how to choose $\mathcal{E}_{\mathrm{m}}$ (and $q$) for a given $\mathcal{E}$...
- ...but we could relate $\mathcal{E}$ and $\mathcal{E}_{\mathrm{m}}$ via a *distinguished limit* (Dormy, 2016)
- Set $\mathcal{E} = \mathcal{E}_0\epsilon$, $\mathcal{E}_{\mathrm{m}} = \mathcal{E}_{\mathrm{m}0}\epsilon^{\alpha}$ for some constants $\mathcal{E}_0$, $\mathcal{E}_{\mathrm{m}0}$, $\alpha$...

# Distinguished limit

- System of equations has four (main) control parameters ($\mathcal{E}$, $\mathcal{E}_{\mathrm{m}}$, $q$, $\widetilde{Ra}$)...
- ...three of which become very small at Earth-like values ($\mathcal{E} \sim 10^{-15}$, $\mathcal{E}_{\mathrm{m}} \sim 10^{-9}$, $q \sim 10^{-5}$)

- Unclear how to choose $\mathcal{E}_{\mathrm{m}}$ (and $q$) for a given $\mathcal{E}$...
- ...but we could relate $\mathcal{E}$ and $\mathcal{E}_{\mathrm{m}}$ via a *distinguished limit* (Dormy, 2016)

- Set $\mathcal{E} = \mathcal{E}_0 \epsilon$, $\mathcal{E}_{\mathrm{m}} = \mathcal{E}_{\mathrm{m}0} \epsilon^{\alpha}$ for some constants $\mathcal{E}_0$, $\mathcal{E}_{\mathrm{m}0}$, $\alpha$...
- ...and take $\epsilon \to 0$

# Distinguished limit

- System of equations has four (main) control parameters ($\mathcal{E}$, $\mathcal{E}_\mathrm{m}$, $q$, $\widetilde{Ra}$)...
- ...three of which become very small at Earth-like values ($\mathcal{E} \sim 10^{-15}$, $\mathcal{E}_\mathrm{m} \sim 10^{-9}$, $q \sim 10^{-5}$)
- Unclear how to choose $\mathcal{E}_\mathrm{m}$ (and $q$) for a given $\mathcal{E}$...
- ...but we could relate $\mathcal{E}$ and $\mathcal{E}_\mathrm{m}$ via a *distinguished limit* (Dormy, 2016)
- Set $\mathcal{E} = \mathcal{E}_0\epsilon$, $\mathcal{E}_\mathrm{m} = \mathcal{E}_{\mathrm{m}0}\epsilon^\alpha$ for some constants $\mathcal{E}_0$, $\mathcal{E}_{\mathrm{m}0}$, $\alpha$...
- ...and take $\epsilon \to 0$
- Choose $\mathcal{E}_0$, $\mathcal{E}_{\mathrm{m}0}$, $\alpha$ (and, ultimately, $q$, $\widetilde{Ra}$) to preserve relevant properties (e.g. correct MAC force balance) of solutions

# Distinguished limit

- System of equations has four (main) control parameters ($\mathcal{E}$, $\mathcal{E}_{\mathrm{m}}$, $q$, $\widetilde{Ra}$)...
- ...three of which become very small at Earth-like values ($\mathcal{E} \sim 10^{-15}$, $\mathcal{E}_{\mathrm{m}} \sim 10^{-9}$, $q \sim 10^{-5}$)

- Unclear how to choose $\mathcal{E}_{\mathrm{m}}$ (and $q$) for a given $\mathcal{E}$...
- ...but we could relate $\mathcal{E}$ and $\mathcal{E}_{\mathrm{m}}$ via a *distinguished limit* (Dormy, 2016)

- Set $\mathcal{E} = \mathcal{E}_0\epsilon$, $\mathcal{E}_{\mathrm{m}} = \mathcal{E}_{\mathrm{m}0}\epsilon^{\alpha}$ for some constants $\mathcal{E}_0$, $\mathcal{E}_{\mathrm{m}0}$, $\alpha$...
- ...and take $\epsilon \to 0$

- Choose $\mathcal{E}_0$, $\mathcal{E}_{\mathrm{m}0}$, $\alpha$ (and, ultimately, $q$, $\widetilde{Ra}$) to preserve relevant properties (e.g. correct MAC force balance) of solutions
- Need to study dependence of such solutions on $\mathcal{E}$ and $\mathcal{E}_{\mathrm{m}}$ to help determine constants

# Some questions to address

Now we'll look at some results on (solenoidal) forces (Teed & Dormy, 2023) and upcoming work on geodynamo branches as $\mathcal{E}$ is lowered (Teed & Dormy, 2025).

# Some questions to address

Now we'll look at some results on (solenoidal) forces (Teed & Dormy, 2023) and upcoming work on geodynamo branches as $\mathcal{E}$ is lowered (Teed & Dormy, 2025).

Key questions to keep in mind:

1. Do solenoidal forces help in identifying the leading order force balance and hierarchy in geodynamo simulations?

# Some questions to address

Now we'll look at some results on (solenoidal) forces (Teed & Dormy, 2023) and upcoming work on geodynamo branches as $\mathcal{E}$ is lowered (Teed & Dormy, 2025).

Key questions to keep in mind:

1. Do solenoidal forces help in identifying the leading order force balance and hierarchy in geodynamo simulations?
2. What regime produces the desired MAC balance relevant to Earth's core dynamics and where is it located in parameter space?

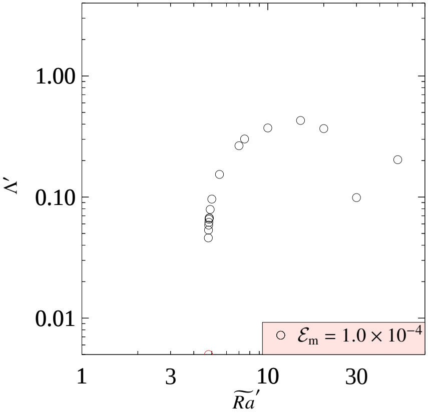
# Some questions to address

Now we'll look at some results on (solenoidal) forces (Teed & Dormy, 2023) and upcoming work on geodynamo branches as $\mathcal{E}$ is lowered (Teed & Dormy, 2025).

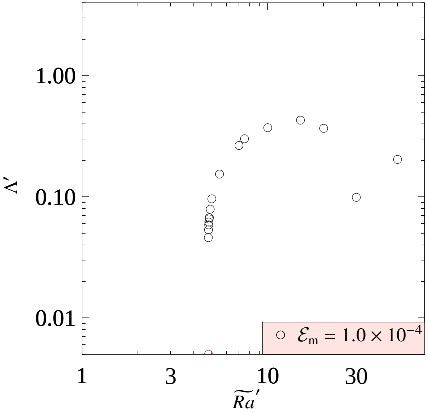
Key questions to keep in mind:

1. Do solenoidal forces help in identifying the leading order force balance and hierarchy in geodynamo simulations?
2. What regime produces the desired MAC balance relevant to Earth's core dynamics and where is it located in parameter space?
3. How does the branching between weak and strong regimes persist/scale as parameters are moved towards Earth-like values? I.e. lowering $\mathcal{E}$ and $\mathcal{E}_{\mathrm{m}}$.

# Branches of dynamo action ($\mathcal{E} = 10^{-4}$)

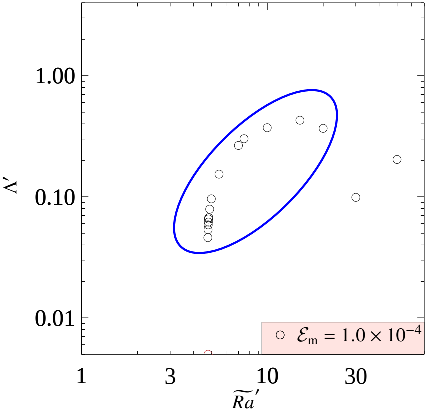


# Branches of dynamo action ($\mathcal{E} = 10^{-4}$)



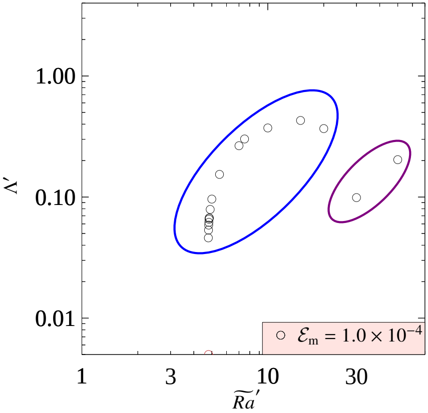

- At higher $\mathcal{E}_{\rm m}$:

# Branches of dynamo action ($\mathcal{E} = 10^{-4}$)



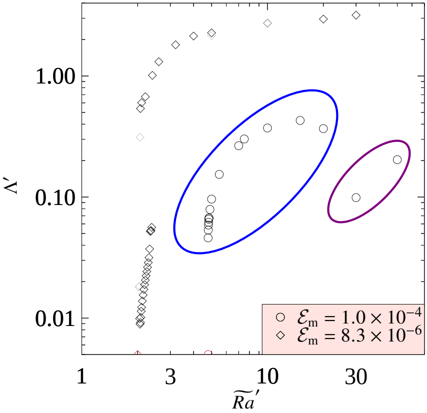

- At higher $\mathcal{E}_m$:
  - Dipolar regime...

# Branches of dynamo action ($\mathcal{E} = 10^{-4}$)



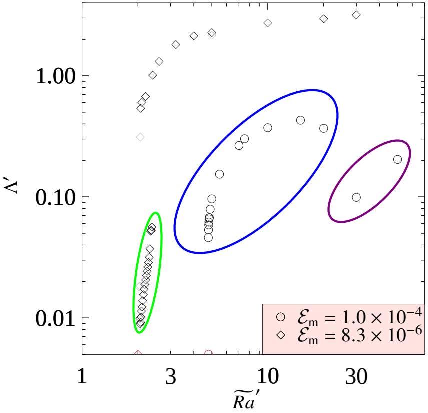

- At higher $\mathcal{E}_\mathrm{m}$:
  - Dipolar regime...
  - ...transitions to multipolar regime at large enough $\widetilde{Ra}$

# Branches of dynamo action ($\mathcal{E} = 10^{-4}$)



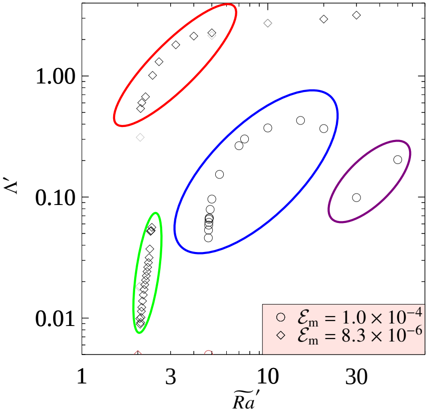

- At higher $\mathcal{E}_\mathrm{m}$:
  - Dipolar regime...
  - ...transitions to multipolar regime at large enough $\widetilde{Ra}$
- At lower $\mathcal{E}_\mathrm{m}$:

# Branches of dynamo action ($\mathcal{E} = 10^{-4}$)



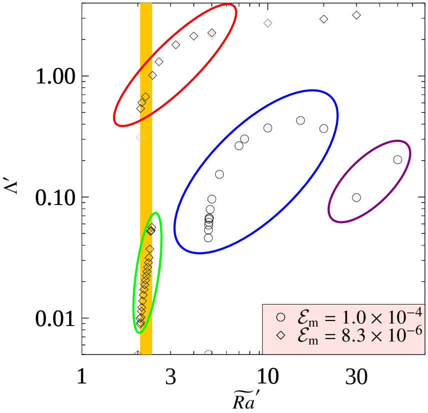

- At higher $\mathcal{E}_\mathrm{m}$:
  - Dipolar regime...
  - ...transitions to multipolar regime at large enough $\widetilde{Ra}$
- At lower $\mathcal{E}_\mathrm{m}$:
  - Weak field dipolar regime ($\Lambda' \ll 1$)...

# Branches of dynamo action ($\mathcal{E} = 10^{-4}$)



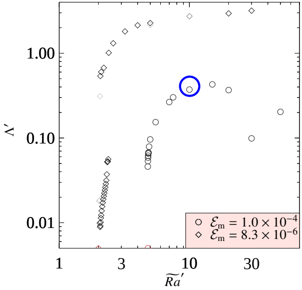

- At higher $\mathcal{E}_\mathrm{m}$:
  - Dipolar regime...
  - ...transitions to multipolar regime at large enough $\widetilde{Ra}$
- At lower $\mathcal{E}_\mathrm{m}$:
  - Weak field dipolar regime ($\Lambda' \ll 1$)...
  - ...transitions to strong field dipolar regime ($\Lambda' \sim 1$)

# Branches of dynamo action ($\mathcal{E} = 10^{-4}$)

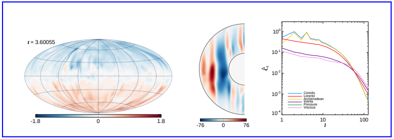



- At higher $\mathcal{E}_m$:
  - ~~Dipolar~~ 'Strongish' dipolar regime...
  - ...transitions to multipolar regime at large enough $\widetilde{Ra}$
- At lower $\mathcal{E}_m$:
  - Weak field dipolar regime ($\Lambda' \ll 1$)...
  - ...transitions to strong field dipolar regime ($\Lambda' \sim 1$)
  - Bistability between weak and strong branches in a region of $Ra$-space

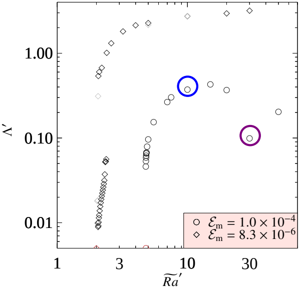
# Typical regimes (at $\mathcal{E} = 10^{-4}$)



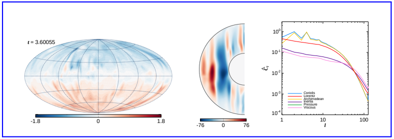

Strongish dipolar



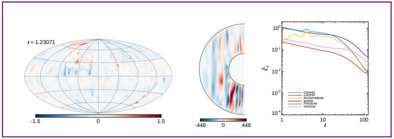
# Typical regimes (at $\mathcal{E} = 10^{-4}$)



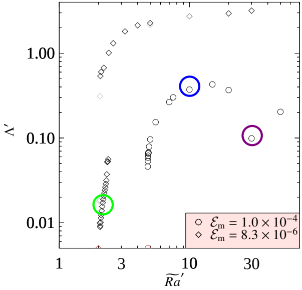

Strongish dipolar



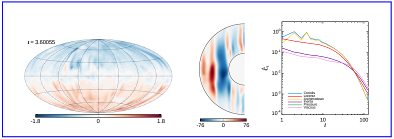

Fluctuating multipolar



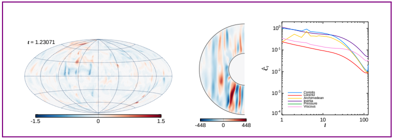
# Typical regimes (at $\mathcal{E} = 10^{-4}$)



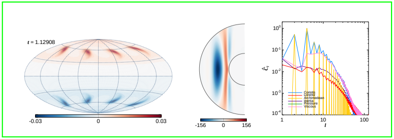

## Strongish dipolar



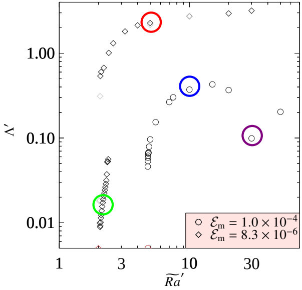

## Fluctuating multipolar



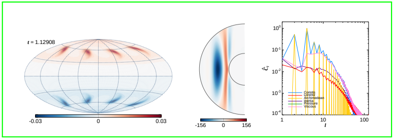

## Weak field dipolar



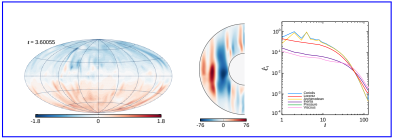
# Typical regimes (at $\mathcal{E} = 10^{-4}$)



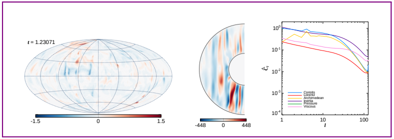

Weak field dipolar



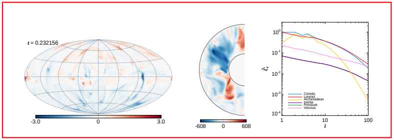

Strongish dipolar




Fluctuating multipolar




Strong field dipolar



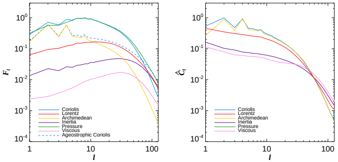
# Comparing forces and solenoidal forces

$$\xrightarrow[\text{Remove gradient parts}]{\text{Perform curl}}$$

# Comparing forces and solenoidal forces

$$\xrightarrow[\text{Remove gradient parts}]{\text{Perform curl}}$$

Strongish dipolar

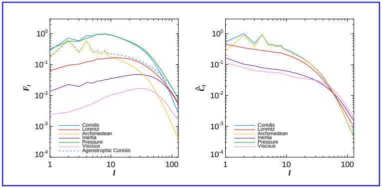



- Forces show mostly QG balance with $\mathbf{F}_{\mathrm{C}}^{\mathrm{ag}} > \mathbf{F}_{\mathrm{C}}$ at some scales. Strange!

# Comparing forces and solenoidal forces

$$\xrightarrow[\text{Remove gradient parts}]{\text{Perform curl}}$$

Strongish dipolar

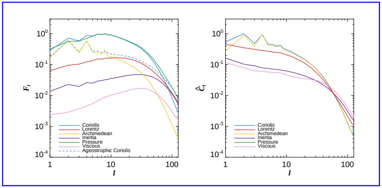



- Forces show mostly QG balance with $\mathbf{F}_C^{ag} > \mathbf{F}_C$ at some scales. Strange!
- Solenoidal forces reveal inertia and viscous forces enter leading order balance

# Comparing forces and solenoidal forces

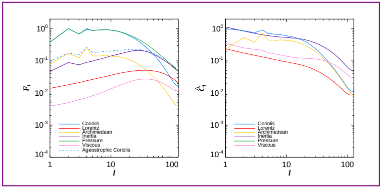
$$\xrightarrow[\text{Remove gradient parts}]{\text{Perform curl}}$$

Strongish dipolar




- Forces show mostly QG balance with $\mathbf{F}_C^{ag} > \mathbf{F}_C$ at some scales. Strange!
- Solenoidal forces reveal inertia and viscous forces enter leading order balance

Fluctuating multipolar

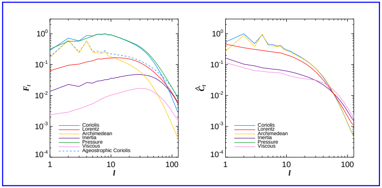



- Forces show inertia entering zeroth order balance

# Comparing forces and solenoidal forces

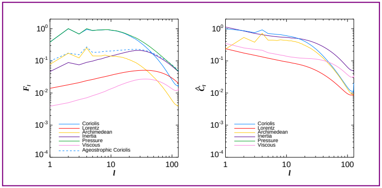
Perform curl
$\longrightarrow$
Remove gradient parts

Strongish dipolar




- Forces show mostly QG balance with $\mathbf{F}_{\text{C}}^{\text{ag}} > \mathbf{F}_{\text{C}}$ at some scales. Strange!
- Solenoidal forces reveal inertia and viscous forces enter leading order balance

Fluctuating multipolar

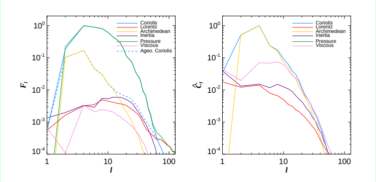



- Forces show inertia entering zeroth order balance
- Solenoidal forces reveal clear leading order CIA balance for multipolar regime

# Comparing forces and solenoidal forces

Perform curl
→
Remove gradient parts

Weak field dipolar

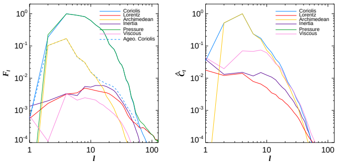



- Forces suggest viscous force unimportant (similar to Lorentz and inertia)

# Comparing forces and solenoidal forces

Perform curl $\longrightarrow$ Remove gradient parts

Weak field dipolar

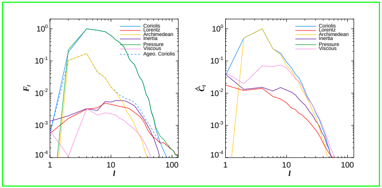



- Forces suggest viscous force unimportant (similar to Lorentz and inertia)
- Solenoidal forces reveal expected leading order VAC balance for weak field regime

# Comparing forces and solenoidal forces

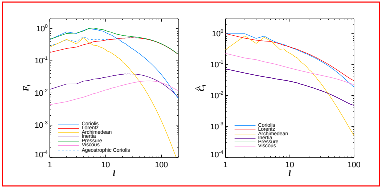
Perform curl $\longrightarrow$ Remove gradient parts

Weak field dipolar




- Forces suggest viscous force unimportant (similar to Lorentz and inertia)
- Solenoidal forces reveal expected leading order VAC balance for weak field regime

Strong field dipolar

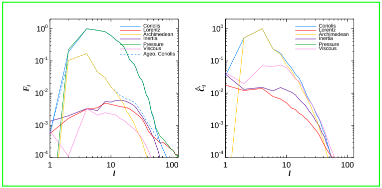



- Forces show ‘QG-MAC’ balance with $\mathbf{F}_{\mathrm{C}}^{\mathrm{ag}} \gg \mathbf{F}_{\mathrm{C}}$ at some scales. Strange!

# Comparing forces and solenoidal forces

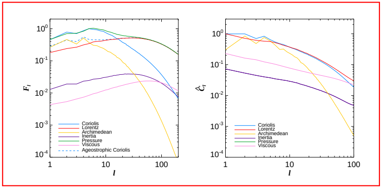
$$\xrightarrow[\text{Remove gradient parts}]{\text{Perform curl}}$$

Weak field dipolar




- Forces suggest viscous force unimportant (similar to Lorentz and inertia)
- Solenoidal forces reveal expected leading order VAC balance for weak field regime

Strong field dipolar

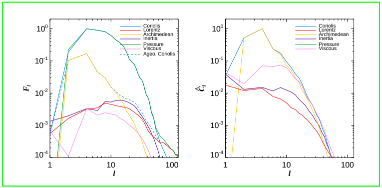



- Forces show 'QG-MAC' balance with $\mathbf{F}_{\mathrm{C}}^{\mathrm{ag}} \gg \mathbf{F}_{\mathrm{C}}$ at some scales. Strange!
- Solenoidal forces reveal clear leading order MAC balance at large scale for strong field regime

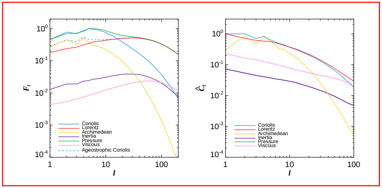
# Comparing forces and solenoidal forces

$$\xrightarrow[\text{Remove gradient parts}]{\text{Perform curl}}$$

Weak field dipolar



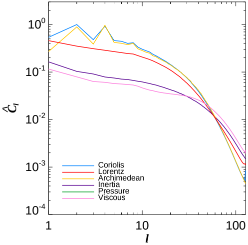

Strong field dipolar



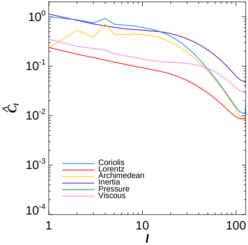

- Forces suggest viscous force unimportant (similar to Lorentz and inertia)
- Solenoidal forces reveal expected leading order VAC balance for weak field regime

- Forces show 'QG-MAC' balance with $\mathbf{F}_\mathrm{C}^{\mathrm{ag}} \gg \mathbf{F}_\mathrm{C}$ at some scales. Strange!
- Solenoidal forces reveal clear leading order MAC balance at large scale for strong field regime
- Usefulness of ageostrophic Coriolis force lost at lengthscales where balance is not QG

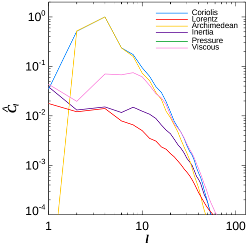
# Solenoidal forces



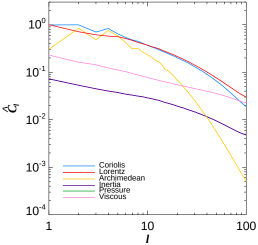

*Strongish*




*Multipolar*




*Weak field*




*Strong field*