

# List of publications cited in the 2019 FDEPS Course

## Convection in solid planetary interiors and implications for their evolution

Stéphane Labrosse, ENS Lyon, France

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### Part I. Fundamentals of Rayleigh-Bénard convection

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